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RELIGIO MATHEMATICI.

PRESIDENTIAL ADDRESS DELIVERED BEFORE THE MATHEMATICAL ASSOCIATION OF AMERICA, SEPTEMBER 7, 1921.

By DAVID EUGENE SMITH, Columbia University.

Apologia. Well aware that I, today, break all precedents in the nature of a presidential address, it is proper that I offer due apology and confess my unconventionality. It is proper, too, that I should frankly say that I am aware that my message is wanting in the rigor of demonstration to which our science accustoms us; that it involves no mathematics beyond the merest elements; that it will be listened to with regret by some and with disapproval by others; and that it is likely to carry conviction to only a limited number.

Why, then, should I choose such a topic? Why should I force my friends to apologize for my thoughts, and why should I place others in a favorable position to condemn my action? The answer is a simple one:—I believe it to be a duty laid upon us, that one of our number should, on some such occasion as this, say what I shall say. I regret that another,—one with more faith, one better endowed with power of expression, one whose words would carry greater weight than any I can utter,—might not have had the opportunity that is mine to deliver the message. And finally, by way of apology, I say with perfect candor that I speak not at all to the audience here present,—for what I shall say will convey nothing that is new to any one of you; I speak rather to those who are not present, those who are not of our special guild, and those to whom the teaching of mathematics may possibly, by this means, appear in a somewhat different, and I hope in a somewhat more favorable light.

The General Message. It is nearly three centuries ago that Sir Thomas Browne wrote, for private circulation among a few of his intimate friends, his *Religio Medici.* He did not call it *Religio Medicorum,* for he could not assume to speak for others. Indeed, as he confessed, the men who practiced the healing art in his time were generally, like loud-mouthed boys in their teens, boastful of their atheism; but for himself, a humble practitioner in an ancient town of England, he could testify; and he, the giver of balm for the body, could unobtrusively pass the message to others, as a balm for the spirit.

And so, today, I speak not of the *Religio Mathematicorum,* for I have not the authority; I feel, indeed, that *Religio Mathematici* is a misnomer; but I also feel that it is proper to speak of the relation of mathematics to a religious attitude of mind, and I know of no better title for my purpose than the one I have chosen.

Do not, however, feel that the message seeks to change the faith or lack of faith of any man; do not feel that it contains a plea for any creed or for any sect; do not feel that it sets forth anything that is new; but rather feel that it represents the mere commonplace knowledge that most of us have, and the mere basis of belief that mathematics may possibly foster. It goes no farther, it seeks only a single corner of the foundation walls upon which we may build, it lacks warmth, it lacks beauty, it lacks the fervor of religious faith, and it is hard of texture; but the foundation stones of palaces, and of temples, and of homes, all lie below the surface; they, too, are cold, but they sustain structures which give to the world pleasure and protection and repose. The message is transmitted by one who has joined in serious conference with those who hold the Brahmin faith; by one who has learned much from the high priests in the temples of Buddha, who has sat at the feet of the followers of Zoroaster, and who has communed with those who wear the green turban that marks the seed of the Prophet,—one who believes that

> "Creed and rite perchance may differ,
> Yet our faith and hope be one,"

and who, with the author of the *Religio Medici* itself, is able to say:

"For my Religion, though there be several Circumstances that might perswade the World I have none at all, . . . yet, in despight hereof, I dare without usurpation assume the honourable Stile of a Christian."

Specifically, I wish to consider, even though totally unable to answer satisfactorily, these questions: What bond of concord, if any, is there between mathematical knowledge and religious faith? What influence can an exact, abstract, reputedly frigid science like ours have upon the religious nature of man? What, in fact, is the soul of mathematics, and to what wave lengths must our own souls be tuned to catch its message?

Such, you will say, are the imaginations of a dreamer; not the serious thoughts of a mathematician. So be it. Were it necessary to make the choice, I would rather be a dreamer without mathematics, than a mathematician without dreams or a teacher without imagination. What I wish to show to those who are not of our calling, however, is that there is no other science that leads so directly to a recognition of the reasonableness of a broad religious faith, and none that parallels so completely the broader tenets of the Fathers.

What is Religion? In the domain of mathematics we find it necessary at certain times and convenient at certain other times to define our terms; but unless we need a particular term in a proof we do not feel bound to define it with precision. For example, in elementary mathematics we do not deem it necessary to define space, number, or straight line, the terms being basic and, for the beginner, indefinable.

So if you ask me to define religion before we consider its relation to mathematics, I must simply refer you to the theologians, wishing you good fortune in your quest. To many people the Buddhist is not religious, but I have often found him intensely and beautifully so. To many religiously minded persons the Parsee is a heathen, but I have seen as great faith and as pronounced goodness of work among the Parsees as I have found among many Christians with whom I worship. For our purposes, therefore, I prefer to think of religion with respect to certain general characteristics, one of which is faith; faith, as a very intellectual and catholic-minded writer has remarked, in "The Eternal, not ourselves, that makes for righteousness"; faith that this life does not end all; faith that our lives today prepare for our lives somewhere beyond.

Tangible Immortality. Out of the manifold characteristics of the mind of youth, one of the most interesting is best described by the word "opinionatedness"—a word that I grant should be in the *Index expurgatorius*. All normal individuals reach this milestone, most of them pass it, but some find their development arrested at this point, and here they bind themselves for life. In particular, they absorb (I will not say develop) the idea that immortality is an idle word of an idle faith, a part of the "opiate of the proletariat," as our most modern autocracy has phrased it.

One thing that mathematics early imparts, unless hindered from so doing, is the idea that here, at last, is an immortality that is seemingly tangible,—the immortality of a mathematical law. The student of algebra, for example, may well question the use of the traditional curriculum, but when he finds the value of $(a + b)^2$ he has come in contact with an eternal law. The laws of the Medes and Persians, unchangeable though they were thought to be, have all perished; the canons that bound Egyptian activities for thousands of years exist only in the ancient records, preserved in our museums of antiquity; the laws of Rome, which at one time dominated the legal world, have given place to modern codes; and the laws that we make today are certain to be changed tomorrow. But in the midst of all these changes it has ever been true, it is true today, it shall be true in all the future of this earth, and it is equally true throughout the universe whether in the algebra of Flatland or in that of the space in which we live, that $(a + b)^2 = a^2 + 2ab + b^2$.

We may change the symbols,—they are temporary expedients to convey the idea; we may speak in different tongues,—they are local expedients to convey thought; but it is inconceivable to us that the relation which the formula expresses should not be true always and everywhere,—a tangible symbol of the immortality of law.

What I learned in chemistry, as a boy, seemed true at the time, but much of it today is known to be false. What I learned of molecular physics seems at the present time like children's stories, interesting but puerile. What we learn in history may be true in some degree, but is certain to be false in many particulars. So we may run the gamut of learning, and nowhere, save in mathematics alone, do we find that which stands as a tangible symbol of the immortality of law, true "yesterday, today, and forever."

But does the teacher make this known to the student? Does the student come to feel the significance of this fact,—this fact so full of awe to the normal mind, this evidence of immortality that never comes to his consciousness until he meets it in mathematics? I do not know, nor do I know how much else that is great, that has tremendous significance, is taught or is not taught in science, in letters, in history, or in art. I only know that mathematics can do this thing; that it can (and it should) give, to the degree that the pupil is able to receive it, the idea that before the world was created, before our solar system was formed, and after our system shall cease to be, the every-day laws of mathematics stood and shall stand for immortal truth,—for laws that are divine in their infinite endurance. It is not necessary, it is not desirable, that we should preach these things in the halls of learning; but it is essential that we should feel their significance. This done and all the rest "shall be added unto you." Stated in another way, the immortality of law means that we come in touch with the invariant. The tyro in mathematics comes early upon the invariant properties of a figure as seen in the theory of elementary projection. In a wider sense, however, all geometry is a science of invariance. We prove a law for a general plane triangle and it never varies, whatever we do to the figure. If we prove that $a^2 = b^2 + c^2 - 2bc \cos A$, then, however A may change, the law itself will never vary. In it the pupil comes into touch with the unchangeable, with the absolute.

It is the same with all other laws of geometry. In any convex polyhedron, whatever its shape, the law remains that the number of faces plus the number of vertices is equal to the number of edges increased by two.

"Change and decay in all around I see," but the established properties of a general geometric figure, in our space, are as unchangeable in that space as divinity itself. Stated in still another way, the immortality of law and the invariability of mathematical principles mean the eternity of mathematics. To come into relation with a science which was illustrated by the spiral nebulae before our solar system was formed, which only now reveals to us those laws of crystals which were in operation long before life appeared upon the earth, and which is also entirely independent of matter, so that if we could imagine the universe destroyed absolutely, the laws would still be true,—to come into relation with such a science makes real to us, as no other discipline in our curriculum can possibly do, the ideal of truth eternal.

Our Infinitesimal Nature. I know of nothing which acts as such a powerful antidote to that which I ventured to call "opinionatedness," as a study of mathematics. To know that the light from solar systems far larger than our own has been thousands of years in reaching us, gives us an idea of our infinitesimal nature, in comparison with space about us, that can come only with a study of the science that it is ours to teach. A bacillus in our veins, so small as to be invisible through a powerful microscope, is a giant compared with ourselves in our relation to this space in which we live. Our doubts, our beliefs, our hopes, our fears are all so trivial, so infinitesimal, so like a lost electron in our solar system, as compared with our relative importance in the universe as revealed to us by the calculations which mathematics brings to bear upon the great problem! Cowper wrote well when he put in verse the words,

> "God never meant that man should scale the heavens
> By strides of human wisdom."

and even the mathematics of youth confirms the thought.

With our feeble voices we cry out that we are certain that this life ends our existence, but mathematics shows us the truth of Voltaire's words, in speaking of human opinion of the significance of life,—

"Doubt is not a pleasant condition, but certainty is an absurd one."

Our feeble voices join in a protest that we cannot understand God. Again the "old man of Ferney," condemned by so many as an atheist, speaks out and says,

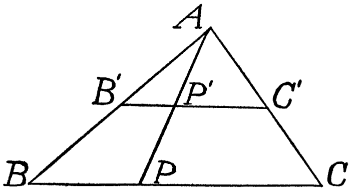
"My reason tells me that God exists; but it also tells me that I cannot know what He is."

The infinitesimal still cries out that it cannot understand what our soul is, and again the sage speaks:

"Is it indeed likely that we should know what our soul is, when we could form no idea of light if we had the misfortune to be born blind?"

We agree that spaces of higher dimensions than the one in which we think we live can easily be conceived by analogy, and we agree without question to the paradoxes which we meet in the study of infinity, and yet we feel that it shows our great wisdom, or perhaps our boldness, if we deny the soul an existence. Strange, that in algebra we accept without the slightest question the idea of the permanence of law, but that our little natures should so often boast that we deny the permanence of the soul!

Contact with the Infinite. One of the impressive experiences which comes to the devotees of our science is the continual contact with the infinite,—an experience which is inspiring beyond words to express,—and sometimes as discouraging. To take an illustration that is a mere commonplace to those who have even visited the borderlands of our science, suppose that a line segment $B'C'$ joins the mid-points of the sides AB and AC of the triangle ABC, and that the straight line $AP'P$ connects the vertex A with points P' and P on $B'C'$ and BC respectively. It is then evident that, in this construction, to any point P on BC there is one point and only one point P' on $B'C'$ that corresponds to it, and conversely. Like it or not as we please, there is only one possible conclusion, namely, that there is a one-to-one correspondence between the points on $B'C'$ and the points on BC; or, taking the usual definition of equality, that there are precisely as many points on $B'C'$ as there are on BC, which is twice as long. But similarly, the number of points on $B'C'$ can be shown to be the same as the number on any part of BC, and so a part (not of a line, but of a group of points) is equal to the whole. To adopt a paradox of old Tertullian, "Certum est quia impossibile est."



Against such a conclusion our little minds revolt; we have been taught, or we have empirically learned, that the part is less than the whole,—as it really is in dealing with the finite. But when we come to deal with the infinite, our little, narrow, finite laws break down; they are even more feeble than the beliefs and experiences of the child, when viewed by the eyes of a mature man. We must face the inevitable, that in the domain of the infinite, the part may indeed be equal to the whole, in spite of the childish beliefs of our finite minds. The mathematician continually meets this necessary conclusion; it is a commonplace; to him the laws of the finite give way without question when he enters the domain of the infinite. In all the finite, indeed, he sees the infinite. He knows that there is a one-to-one correspondence between the points in a sphere and those in the entire universe about him, and between the number of vibrating points in his brain and the number in all space. In other words, within the brain of each of us, there is a point that corresponds to any given point in the universe, say one on the surface of Neptune. If we take another point, say the center of a fixed star, there is one special point, and only one, within each brain that also corresponds to that. If we move the point in the fixed star ever so slightly, we move the point in the brain so that it shall continue to correspond to it. Does all this signify anything to us? I certainly do not know. Does it mean that the planets have their influence upon us? I do not know, but with my finite mind I am led to say I do not think so. Does this mathematico-physical relation of our brains to all the universe about us have any deeper significance? I do not know. But one thing I do know, that thoughts like these give new meaning to the words, "For, behold, the Kingdom of God is within you."

Do such ideas signify that it is ours to preach? Shall mathematics become a medium for religious instruction? Do we not lower religious belief when we link it to the certainties of cold mathematics? With respect to each of these questions I would reply, "I do not think so." With respect to the last one, I see no difference in the sanctity of truth, whether the truth be taught in the books of Euclid, in the holy books of the East, or in the Christian Church;—in the *significance* of truth,—yes; in the *sanctity* of truth,—no.

Our Impotence in Relation to the Eternal. I know of no other branch of learning that makes so clear to us our impotence in relation to the Eternal. We fail today in a problem in chemistry, but we feel that we may succeed tomorrow; we are at a loss to know how to overcome a certain difficulty in physics, but we have confidence that someone, sometime, somehow, will overcome it; we do not attain to the success we had hoped for in the painting of a certain picture, but we "carry on" in the hope of bettering our work from day to day; and so in mathematics, we fail but we persevere. But there come times in mathematics when we fail and know that we must fail, because we come in conflict with the Eternal. At first sight we say that we can construct a seven-edged polyhedron. We fail, we seek out the reason, and we find that we are combatting the everlasting truth of which I have spoken, the truth that $f + v = e + 2$. Protest as we will, we are powerless when we combat the Eternal.

Physical Permanence. Our mathematics also comes to the aid of science and assists in the proof that force is not lost, and shows us that even a thought generates a wave which has its eternal influence. I like the ancient theory, and I know of no reason why it should not be a fact, that the very walls and arches of the venerable cathedrals of the old world, which have heard the daily chants of the priests for many centuries, have come to vibrate in unison therewith, so that the same singers, if transported to another spot, fail to produce the same sonorous, musical effect. Why, then, should not the thoughts of others influence us, as the telepathists affirm, and why should not our body of thought be permanent in space, even from a materialistic point of view?

Mathematics is such a science of harmony! The universe is such a science of harmony! The "music of the spheres" meant precisely this in the ancient philosophy;—and how out of all harmony with what mathematics reveals to us is the theory of annihilation, even, as I have said, from the materialistic standpoint!

To quote again from Voltaire, whose name is anathema to so many ignorant religionists:

"All nature cries aloud that He *does* exist; that there *is* a supreme intelligence, an immense power, an admirable order, and that everything teaches us our own dependence upon it."

Spoken like a philosopher, but also like the mathematician that he was; for Voltaire was a great student of Newtonian philosophy and did more than any man in his century to make the teachings of Newton known to the general intellectual element of France. That Voltaire, the bitter antagonist of fraud and sham, in the Church as well as in the State, should honestly confess his faith in this manner need not surprise us. It shows his greatness. Cicero told the world two thousand years ago that the greatest thinkers always have such faith:—

"There is, I know not how, in the minds of men a certain presage, as it were, of a future existence; and this takes the deepest root and is most discoverable in the greatest geniuses and the most exalted souls."

The Drama of Space. What a science is ours that raises the curtain on the drama of space! That shows us a finite space in which all bodies are at rest until acted upon by some external force; and then the space of the infinitesimal, in which a radically different code of laws obtains,—where everything (molecules, atoms, electrons, and very likely sub-electrons) is an automaton, self moved and never resting. John Burroughs has expressed this idea, adding:

"When we reach the astronomic world, or the sidereal universe, we find the same condition that prevails in the world of the infinitely little: perpetual motion goes on, friction is abolished, and nothing is at rest. . . . Height and depth, upper and under, east and west, north and south, weight and inertia, as we experience them, have vanished. There are no boundaries, no ending and no beginning, no center and no circumference; the infinite cannot have any of these."

What a science is ours, moreover, that reveals to the youth the secret of indirect measurement! That shows him how the distances to the stars are found! That opens to him the moving picture of the universe! That reveals the world of the electronic bodies as the reciprocal of the world of the sidereal bodies! And who can measure the influence of this revelation upon the soul of one who is standing upon the threshold of young manhood or young womanhood?

Scientific Religion. Religion is generally felt to be unscientific. One of the world's great authorities, Professor Harald Höffding of Copenhagen, in his *Philosophy of Religion*, speaks of it as that state of mind "in which feeling and need, fear and hope, enthusiasm and surrender play a greater part than do meditation and inquiry, and in which intuition and imagination have the mastery over investigation and reflection."

I cannot believe that this should be the case with respect to the great basal facts of religion, although it is so traditionally, at least in our sectarian doctrines. We lay down certain postulates in geometry; they may, as in the case of parallels, be true or false; mathematics simply says, "If A, then B,"—if these postulates are sound, then these conclusions are true. I have often wondered mildly why religion did not do the same, postulating certain statements and then proceeding precisely as in mathematics,—"If A, then B." The postulates, like those of Euclid, might be true or false, but the deductions would be absolute. I have no idea as to what postulates would be assumed; I simply know that it seems entirely scientific to assume at least a few that are reasonable. We do not, in elementary mathematics, feel that our postulates must be independent or that they must cover all possible needs; we simply assume what seems to be necessary for the mind of youth, and on that we build.

When I think, in a kind of indefinite fashion, of what my mind postulates with respect to some of the larger features of mathematics, my thought runs to something like this:

1. The Infinite exists.
2. Immortal laws exist.
3. The laws relating to finite magnitudes do not hold respecting the infinitely large or the infinitely small.
4. The existence of hyperspace is entirely reasonable.
5. No factor is ever lost.
6. Time may be a closed curve.

Such a list of postulates might easily be put into theological language as well, and might be extended when necessary. For example, the theologian might phrase these same postulates like this:

1. God exists.
2. God's laws exist.
3. God's laws are entirely different from ours.
4. There are spaces beyond ours.
5. The soul exists and is eternal.
6. God looks at time as a whole.

After making out his list of postulates, the theologian might formulate his definitions. It would be as easy, I should think, to make an attempt to define God, heaven, angel, and miracle as it is to make an attempt to define infinity, hyperspace, straight line, solid angle, and dozens of other terms that we use in mathematics; or, if precise definition were found unnecessary, as is the case with these mathematical terms, at least some reasonable limitations would be in order.

Given the postulates and the definitions, I see no reason why a perfectly rigorous set of propositions should not be erected and religion put on a cold, scientific foundation. I should not wish to see this done; I think it would be about as sensible as to build up a scientific, deductive system of love or beauty; but what I mean to say is that if religion is unscientific, it is partly because the world wishes it to be so. Love would not be any more potent, or more real, or more beautiful if we formulated a set of postulates and deduced a series of propositions relating to it; and the same may be said of religion.

Duality of Mathematics and Religion. Schopenhauer's duality between Time and Space is well known. Time is homogeneous, for example; it is a continuum, and no part is separated from any other part by something which is not time. The dual proposition for Euclidean space is simply formed by a substitution of "space" for "time."

We need not be surprised at this dualism. Time has often been called a fourth dimension, and it harmonizes with our three dimensional, Euclidean space to think of time alone as a space of one dimension. The two united gave to Sir William Rowan Hamilton the science of quaternions, and now they unite again to give birth to one feature of the Minkowski-Einstein hypothesis.

How much of Schopenhauer's duality is real we cannot say, because we cannot say how much of time and space are real. If we leave the domain of Euclidean space and think of ourselves as living in a space that curves through a fourth dimension, as a spherical two-dimensional space curves through a third dimension, then this space becomes limited, as Einstein suggests, and we are led to consider other three-dimensional spaces like ours,—all quasi-finite, and the Schopenhauer dualism automatically changes.

Similarly, when we consider the duality between mathematics and religion, we have nothing positive. Change the space in which we live, make time a fourth dimension, let our new universe curve through a fifth dimension, and the details of the parallelism would necessarily change. Grant, however, certain postulates, derived empirically like the postulates of mathematics, or, as we say, derived from common sense,—grant these, and some such parallelism as the following is suggested to the mathematical mind:

MATHEMATICS.	RELIGION.
1. The infinite exists.	1. God exists.
2. Eternal laws exist.	2. Eternal laws exist.
3. The laws relating to finite magnitudes do not hold respecting the infinitely large or the infinitely small.	3. God's laws are so different from ours as to be absolutely non-understandable by us.
4. The existence of hyperspace is entirely reasonable.	4. The existence of a heaven, with gradations, is entirely reasonable.
5. No factor is ever lost.	5. The soul is eternal.
6. Time may be a closed curve.	6. God looks at time as a whole.
7. Time may be a fourth dimension.	7. In the next world, the direction of time may actually be seen.

8. Positive infinity may physically coincide with negative infinity, if lines curve through four-space.	8. In God's sight the infinite past and the infinite future are the same.
9. A Flatlander has enough of the third dimension in his being to give him some feeling of that dimension; and so this may explain the fact that we have some feeling of the fourth dimension.	9. The human soul has enough of the divine within it to have some feeling of the reality of divinity and of the world beyond.
10. Mathematics is a vast storehouse of the discoveries of the human intellect. We cannot afford to discard this material.	10. Religion is a vast storehouse of the discoveries of the human spirit. We cannot afford to discard this material.
11. It is not necessary that the solution of a problem, by limited means,—say the trisection of an angle,—should be found in order that we may feel certain that the problem can be solved by *some* means.	11. It is not necessary that the solution of the problem of religion, by our limited human means, should be found in order that we may feel certain that the problem can be solved by *some* means.
12. Every term in an infinite sequence is in a small way a part of infinity.	12. Lucretius spoke wisely when he said, "Everyone is in a small way the image of God."

Conclusion. And what is the conclusion? Does mathematics make a man religious? Does it give him a basis for ethics? Will the individual love his fellow man more certainly because of the square on the hypotenuse?—Such questions are trivial; they are food for the youthful paragrapher. Mathematics makes no such claim. What we may safely assert, however, is this,—that mathematics increases the faith of a man who has faith; that it shows him his finite nature with respect to the Infinite; that it puts him in touch with immortality in the form of mathematical laws that are eternal; and that it shows him the futility of setting up his childish arrogance of disbelief in that which he cannot see.

And if this be the case, then what is the duty of teachers of our science? To preach?—that should be the last thought. The greatest sermons are preached in silence. The most ancient religions that we have, if there be more than one fundamental religion, have always recognized this fact. And so it must be with us,—that we should teach "the science venerable" not merely for its technique; not solely for this little group of laws or that; not only for a body of unrelated propositions or for some examination set by the schools; but that we should teach it primarily for the beauty of the discipline, for "the music of the spheres," and for the faith that it gives in truth, in eternal law, in the Infinite, and in the reality of the imaginary; and for the feeling of humility that results from our comparison of the laws within our reach and those which obtain in the transfinite domain. With such a spirit to guide us, what teachers we would be!—whether of those who are standing on the threshold, of those who are passing through the realms of mystery that lead to manhood and womanhood, of those of mature years, or of those who, as the ancients were wont to say, "number their years upon their right hand." For then, unconsciously but none the less surely, would we prove the words of a seer among poets:

> "And Reason now, through number,
>   time, and space
> Darts the keen lustre of her serious eye;
> And learns, from facts compared,
>   the laws to trace
> Whose long procession leads to Deity."

MATHEMATICAL ASSOCIATION OF AMERICA.

The sixth annual meeting of the Association is to be held at Toronto December 29–30, 1921. The American Association for the Advancement of Science and the American Mathematical Society are to hold their annual meetings there during the same week.

The following 49 persons and 1 institution, on applications duly certified, have been elected to membership.

G. H. Albright, A.M. (Harvard). Prof., Colorado College, Colorado Springs, Colo.
Thurman Andrew, B.S. (W. Va. Wesleyan). Teacher, Wesleyan College, Buckhannon, W. Va.
W. F. C. Arndt, Ph.D. (Göttingen). Prof., Grey Univ. Coll., Bloemfontein, South Africa.
E. A. Bailey, A.B. (Georgia). Instr., U. S. Naval Academy, Annapolis, Md.
W. W. R. Ball, M.A. Trinity College, Cambridge, Eng.
E. B. Beatty, A.M. (California). Asso. prof., Oregon Agric. Coll., Corvallis, Ore.
B. C. Bellamy, B.S.C.E. (Wyoming). Civil Engineer, Laramie, Wyo.
O. J. Bond, LL.D. (South Carolina). Supt., The Citadel, Milit. Coll. of South Carolina, Charleston, S. C.
B. H. Camp, Ph.D. (Yale). Prof., Wesleyan Univ., Middletown, Conn.
C. H. Chepmell, Major, Sussex, Eng.
Carrie A. Day (Fredonia State Normal). Teacher, H. S., Smethport, Pa.
L. G. DuPasquier, Ph.D. (Zurich). Prof. ord., Univ. Neuchatel, Switzerland.
H. W. English. Master in Chancery, Jacksonville, Ill.
Henri Fehr, Docteur ès sciences (Geneva). Prof., Univ. of Geneva, Switzerland.
Philip Fitch, A.B. (Colorado Coll.). Instr., physics, N. Side H. S., Denver, Colo.
A. R. Forsyth, Sc.D., LL.D., Math.D. Chief prof. of math., Imperial Coll. of Sc. and Tech., S. Kensington, London, Eng.
Torwald Fredericksen, Ph.B. (Chicago). Instr., Colgate Univ., Hamilton, N. Y.
Guido Fubini. Prof., Univ. and Polytech. School of Turin, Turin, Italy.
J. C. Funk, A.M. (Columbia). Head of dept. of math., H. S. and Junior Coll., Santa Maria, Calif.
C. L. Garner, B.E. (N. C. State Coll.). Hydrogr. and geod. engr., U. S. Coast and Geod. Survey, Washington, D. C.
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